EE-580 Space Mechanisms

# Exercise 1.2 - Density of gas molecules

v.02

## Problem Statement

From the altitude profile of the atmospheric pressure or from its specific mass, calculate the density of gas molecules (number of molecules per unit volume) and pressure in the environment for the following altitudes:

1. Sea level
2. Karman Line (100km)
3. Low Earth Orbit (360km)

## Hints

The standard atmospheric model is described in ECSS-E-ST-10-04C Rev.1. Use the low solar activity values.
It is considered that the ideal gas law applies, even at low altitude.

## Solution

### Ideal gas

(1) Ideal gas law (for n moles of gas): $p \cdot V = n \cdot N_A \cdot k \cdot T$
With:

- Avogadro constant $[\text{mole}^{-1}]$:

In[ ]:=
```
N_A = 6.022 × 10^23;
```

- Boltzmann constant [J/K]:

In[ ]:=
```
k = 1.38 × 10^-23;
```

### Molar density

For $n$ moles of gas at a temperature $T$ and a pressure $p$, from (1):
(2) $\frac{p \cdot V}{n \cdot T} = N_A \cdot k$

We define the molar density as [molecules/m$^3$]:

(3) $\rho = \frac{n \cdot N_A}{V}$

Substituting $n$ from (2) in (3):

(4) $\rho = \frac{p}{T \cdot k}$

## Specific mass of the gas

With the molar mass of dry air [kg/mole]:

```
In[•]:= m_air = 28.96 × 10^-3;
```

Specific mass [kg/m$^3$]:

$M = \frac{\rho}{N_A} \cdot m_{air}$

Hence the molar density [molecules/m$^3$]:

```
In[•]:= ρ[M_] = M N_A / m_air;
```

And the pressure (from (4) ) [Pa]:

```
In[•]:= p[ρ_, T_] = ρ T k;
```

## Calculations for various altitudes

Specific masses at altitudes 0, 100km, 360km [kg/m$^3$], from ECSS-E-ST-10-04C Rev.1 table G-1:

```
In[•]:= M_0 = 1.07;
        M_100 = 6.18 × 10^-7;
        M_360 = 1.47 × 10^-12;
```

Temperatures at altitudes 0, 100km, 360km [K], from ECSS-E-ST-10-04C Rev.1 table G-1:

```
In[•]:= T_0 = 300;
        T_100 = 171;
        T_360 = 696;
```

Molar densities at altitudes 0, 100km, 360km [molecules/m$^3$]

```
In[•]:= ρ[M_0]
Out[•]=
        2.22498 × 10^25

In[•]:= ρ[M_100]
Out[•]=
        1.28508 × 10^19

In[•]:= ρ[M_360]
Out[•]=
        3.05675 × 10^13
```

Pressure at altitudes 0, 100km, 360km [hPa]

In[•]:= p[ρ[M0], T0] / 100

Out[•]=

921.141

In[•]:= p[ρ[M100], T100] / 100

Out[•]=

0.000303254

In[•]:= p[ρ[M360], T360] / 100

Out[•]=

$2.93594 \times 10^{-9}$

## G.9 Tables

**Table G-1: Altitude profiles of the atmosphere constituents $N_2$, O, $O_2$, He, Ar, H, N and anomalous O for low solar and geomagnetic activities (NRLMSISE-00 model - $F10.7 = F10.7_{avg} = 65$, $A_p = 0$)**

| H (km) | $n_{HE}$ ($m^{-3}$) | $n_O$ ($m^{-3}$) | $n_{N2}$ ($m^{-3}$) | $n_{O2}$ ($m^{-3}$) | $n_{AR}$ ($m^{-3}$) | $n_H$ ($m^{-3}$) | $n_N$ ($m^{-3}$) | $n_{Anomal\ O}$ ($m^{-3}$) | T(K) | ϱ ($kg.m^{-3}$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1,17E+20 | 0,00E+00 | 1,74E+25 | 4,67E+24 | 2,08E+23 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 3,00E+02 | 1,07E+00 |
| 20 | 9,46E+18 | 0,00E+00 | 1,41E+24 | 3,78E+23 | 1,69E+22 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,06E+02 | 8,68E-02 |
| 40 | 4,05E+17 | 0,00E+00 | 6,04E+22 | 1,62E+22 | 7,23E+20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,57E+02 | 3,72E-03 |
| 60 | 3,29E+16 | 0,00E+00 | 4,90E+21 | 1,31E+21 | 5,86E+19 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,45E+02 | 3,01E-04 |
| 80 | 1,89E+15 | 3,01E+15 | 2,73E+20 | 7,27E+19 | 3,25E+18 | 2,65E+13 | 6,64E+10 | 2,28E-51 | 2,06E+02 | 1,68E-05 |
| 100 | 1,17E+14 | 4,78E+17 | 1,02E+19 | 2,38E+18 | 1,04E+17 | 2,70E+13 | 3,10E+11 | 2,24E-37 | 1,71E+02 | 6,18E-07 |
| 120 | 2,50E+13 | 7,23E+16 | 3,11E+17 | 4,36E+16 | 1,36E+15 | 6,07E+12 | 1,19E+12 | 1,41E-27 | 3,53E+02 | 1,88E-08 |
| 140 | 1,50E+13 | 2,12E+16 | 4,89E+16 | 4,45E+15 | 1,09E+14 | 2,17E+12 | 6,16E+12 | 2,44E-19 | 5,21E+02 | 3,08E-09 |
| 160 | 1,16E+13 | 9,37E+15 | 1,38E+16 | 1,06E+15 | 1,88E+13 | 1,09E+12 | 1,61E+13 | 1,12E-12 | 6,05E+02 | 9,49E-10 |
| 180 | 9,61E+12 | 4,88E+15 | 4,76E+15 | 3,34E+14 | 4,23E+12 | 7,31E+11 | 2,20E+13 | 1,45E-07 | 6,48E+02 | 3,70E-10 |
| 200 | 8,21E+12 | 2,73E+15 | 1,80E+15 | 1,15E+14 | 1,08E+12 | 5,94E+11 | 2,02E+13 | 1,20E-03 | 6,70E+02 | 1,63E-10 |
| 220 | 7,12E+12 | 1,59E+15 | 7,14E+14 | 4,11E+13 | 2,98E+11 | 5,32E+11 | 1,51E+13 | 1,22E+00 | 6,82E+02 | 7,80E-11 |
| 240 | 6,21E+12 | 9,42E+14 | 2,93E+14 | 1,51E+13 | 8,62E+10 | 4,98E+11 | 1,04E+13 | 2,43E+02 | 6,88E+02 | 3,97E-11 |
| 260 | 5,45E+12 | 5,66E+14 | 1,23E+14 | 5,66E+12 | 2,59E+10 | 4,75E+11 | 6,85E+12 | 1,40E+04 | 6,92E+02 | 2,13E-11 |
| 280 | 4,78E+12 | 3,44E+14 | 5,27E+13 | 2,16E+12 | 8,02E+09 | 4,57E+11 | 4,50E+12 | 3,08E+05 | 6,94E+02 | 1,18E-11 |
| 300 | 4,21E+12 | 2,10E+14 | 2,30E+13 | 8,42E+11 | 2,54E+09 | 4,41E+11 | 2,96E+12 | 3,26E+06 | 6,95E+02 | 6,80E-12 |
| 320 | 3,71E+12 | 1,30E+14 | 1,01E+13 | 3,33E+11 | 8,22E+08 | 4,27E+11 | 1,96E+12 | 1,96E+07 | 6,96E+02 | 4,01E-12 |
| 340 | 3,28E+12 | 8,05E+13 | 4,54E+12 | 1,33E+11 | 2,71E+08 | 4,13E+11 | 1,30E+12 | 7,64E+07 | 6,96E+02 | 2,41E-12 |
| 360 | 2,90E+12 | 5,02E+13 | 2,06E+12 | 5,42E+10 | 9,06E+07 | 4,00E+11 | 8,74E+11 | 2,13E+08 | 6,96E+02 | 1,47E-12 |
| 380 | 2,56E+12 | 3,15E+13 | 9,43E+11 | 2,23E+10 | 3,07E+07 | 3,87E+11 | 5,88E+11 | 4,60E+08 | 6,96E+02 | 9,14E-13 |
| 400 | 2,27E+12 | 1,99E+13 | 4,37E+11 | 9,29E+09 | 1,06E+07 | 3,75E+11 | 3,98E+11 | 8,15E+08 | 6,96E+02 | 5,75E-13 |
| 420 | 2,01E+12 | 1,26E+13 | 2,04E+11 | 3,91E+09 | 3,66E+06 | 3,64E+11 | 2,70E+11 | 1,24E+09 | 6,96E+02 | 3,66E-13 |
| 440 | 1,78E+12 | 8,06E+12 | 9,61E+10 | 1,66E+09 | 1,29E+06 | 3,53E+11 | 1,85E+11 | 1,69E+09 | 6,96E+02 | 2,35E-13 |
| 460 | 1,58E+12 | 5,17E+12 | 4,56E+10 | 7,13E+08 | 4,55E+05 | 3,42E+11 | 1,26E+11 | 2,09E+09 | 6,96E+02 | 1,53E-13 |
| 480 | 1,41E+12 | 3,33E+12 | 2,18E+10 | 3,09E+08 | 1,63E+05 | 3,32E+11 | 8,68E+10 | 2,42E+09 | 6,96E+02 | 1,01E-13 |
| 500 | 1,25E+12 | 2,15E+12 | 1,05E+10 | 1,35E+08 | 5,87E+04 | 3,22E+11 | 5,99E+10 | 2,66E+09 | 6,96E+02 | 6,79E-14 |
| 520 | 1,11E+12 | 1,40E+12 | 5,11E+09 | 5,92E+07 | 2,13E+04 | 3,12E+11 | 4,15E+10 | 2,81E+09 | 6,96E+02 | 4,63E-14 |
| 540 | 9,91E+11 | 9,14E+11 | 2,50E+09 | 2,62E+07 | 7,81E+03 | 3,03E+11 | 2,88E+10 | 2,88E+09 | 6,96E+02 | 3,21E-14 |
| 560 | 8,83E+11 | 5,99E+11 | 1,23E+09 | 1,17E+07 | 2,88E+03 | 2,94E+11 | 2,00E+10 | 2,88E+09 | 6,96E+02 | 2,28E-14 |
| 580 | 7,88E+11 | 3,94E+11 | 6,07E+08 | 5,24E+06 | 1,07E+03 | 2,85E+11 | 1,40E+10 | 2,83E+09 | 6,96E+02 | 1,65E-14 |
| 600 | 7,04E+11 | 2,60E+11 | 3,02E+08 | 2,36E+06 | 4,01E+02 | 2,77E+11 | 9,79E+09 | 2,74E+09 | 6,96E+02 | 1,23E-14 |
| 620 | 6,29E+11 | 1,73E+11 | 1,51E+08 | 1,07E+06 | 1,51E+02 | 2,69E+11 | 6,88E+09 | 2,63E+09 | 6,96E+02 | 9,37E-15 |
| 640 | 5,63E+11 | 1,15E+11 | 7,59E+07 | 4,90E+05 | 5,74E+01 | 2,61E+11 | 4,84E+09 | 2,50E+09 | 6,96E+02 | 7,33E-15 |
| 660 | 5,04E+11 | 7,67E+10 | 3,84E+07 | 2,25E+05 | 2,19E+01 | 2,54E+11 | 3,42E+09 | 2,37E+09 | 6,96E+02 | 5,88E-15 |
| 680 | 4,51E+11 | 5,14E+10 | 1,95E+07 | 1,04E+05 | 8,42E+00 | 2,46E+11 | 2,42E+09 | 2,23E+09 | 6,96E+02 | 4,83E-15 |
| 700 | 4,04E+11 | 3,45E+10 | 9,94E+06 | 4,84E+04 | 3,26E+00 | 2,39E+11 | 1,72E+09 | 2,09E+09 | 6,96E+02 | 4,04E-15 |
| 720 | 3,63E+11 | 2,33E+10 | 5,10E+06 | 2,26E+04 | 1,27E+00 | 2,33E+11 | 1,22E+09 | 1,96E+09 | 6,96E+02 | 3,44E-15 |
| 740 | 3,26E+11 | 1,58E+10 | 2,63E+06 | 1,06E+04 | 4,97E-01 | 2,26E+11 | 8,70E+08 | 1,83E+09 | 6,96E+02 | 2,98E-15 |
| 760 | 2,93E+11 | 1,07E+10 | 1,36E+06 | 5,00E+03 | 1,96E-01 | 2,20E+11 | 6,21E+08 | 1,70E+09 | 6,96E+02 | 2,61E-15 |
| 780 | 2,63E+11 | 7,28E+09 | 7,06E+05 | 2,37E+03 | 7,75E-02 | 2,14E+11 | 4,45E+08 | 1,59E+09 | 6,96E+02 | 2,31E-15 |
| 800 | 2,37E+11 | 4,97E+09 | 3,68E+05 | 1,13E+03 | 3,09E-02 | 2,08E+11 | 3,20E+08 | 1,48E+09 | 6,96E+02 | 2,06E-15 |
| 820 | 2,13E+11 | 3,40E+09 | 1,93E+05 | 5,41E+02 | 1,24E-02 | 2,02E+11 | 2,30E+08 | 1,38E+09 | 6,96E+02 | 1,85E-15 |
| 840 | 1,92E+11 | 2,33E+09 | 1,02E+05 | 2,60E+02 | 4,99E-03 | 1,97E+11 | 1,66E+08 | 1,28E+09 | 6,96E+02 | 1,67E-15 |
| 860 | 1,73E+11 | 1,61E+09 | 5,36E+04 | 1,26E+02 | 2,02E-03 | 1,91E+11 | 1,20E+08 | 1,19E+09 | 6,96E+02 | 1,51E-15 |
| 880 | 1,57E+11 | 1,11E+09 | 2,85E+04 | 6,09E+01 | 8,25E-04 | 1,86E+11 | 8,68E+07 | 1,11E+09 | 6,96E+02 | 1,38E-15 |
| 900 | 1,41E+11 | 7,67E+08 | 1,52E+04 | 2,97E+01 | 3,38E-04 | 1,81E+11 | 6,30E+07 | 1,03E+09 | 6,96E+02 | 1,26E-15 |